Radiation-Induced Acoustic Waves in Water

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Introduction

THE momentum transfer caused by a pulse laser upon a surface has been studied both in vacuum and under atmospheric conditions.[1,2] These studies were concerned primarily with surfaces of solids, i.e., aluminum, tungsten, etc. When a water surface is irradiated by a powerful pulse laser, an acoustic wave has been observed to propagate into the water.[3-5] These experimental observations were made with CO_2 lasers and with small spot size (diameter $d \lesssim 1$ cm). Here we will consider $d \sim 30$ cm. The phenomenology of laser induced acoustic waves in water is rather complex. For very high intensity laser beams, a plasma will be generated above the surface and will alter the characteristics of the beams. For simplicity, when the water is vaporized, the droplets in the water vapor above the surface are assumed to be of the same size as the average aerosols, i.e., 3-5 μm.[6] From the summary of the observed breakdown data in air,[7] the plasma formation threshold for 3-5 μm particles is of the order of 10^8 W/cm^2. Since the formation of plasma is an inefficient mechanism in the process of radiation induced sound, we consider only laser intensities $\leq 10^8$ W/cm^2.

In order to assess the acoustic signals in water, we must first establish the pressure history on the surface. Here, we will divide the acoustic signal generation into two regimes according to laser intensity. For high intensities, i.e., $10^7 - 10^8$ W/cm^2, the blast wave theory will be used to give the surface pressure history. In applying this theory it is assumed that the laser energy is completely absorbed in the blast wave, and in addition, we can ignore the presence of water vapor which may be significant. For low intensities, i.e., less than 10^6 W/cm^2, a previously developed thermal conduction code[3] will be used to predict the pressure history on the surface. Finally, knowing the surface pressure history, we can proceed to estimate the far-field signatures.

Acoustic Source

High Intensity Regime

The classical blast wave theory generally refers to the propagation of shock waves in a gaseous medium due to an instantaneous energy input. Since we are interested in the pressure on the surface at all times, it is more appropriate to use the variable energy blast wave theory while the laser is on.[8] Consider the spot size diameter as $d = 30.48$ cm, the laser pulse as having a top hat profile in both time and space, and the wave expansion being planar. The planar assumption will be justified later by the solution. The top hat profile assumption requires that the energy input be a linear function of t, $E = It$, where I is the laser intensity. Then the shock radius R can be expressed as:

$$R = (Ia_0^2/p_0 J_0)^{1/3}\, t$$

where a_0 is the speed of sound; p is the pressure; J_0 is an integral given in Fig. 1 of Ref. 7; and the subscript 0 indicates conditions in the undisturbed medium. With the above linear energy input, we get $J_0 = 1$. In addition, when we consider that $I = 10^8$ W/cm^2; $\tau = 10^{-6}$ sec; $d = 30.48$ cm; and the

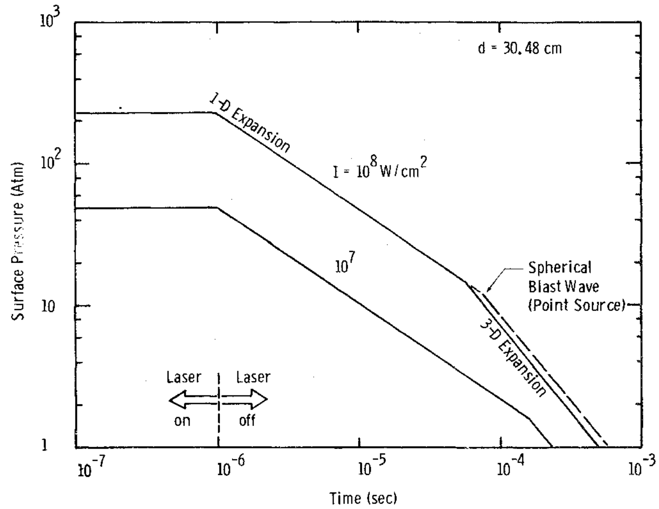

Fig. 1 Surface pressure as function of time for high intensity laser pulses.

undisturbed medium is the sea level atmosphere ($p_0 = 1$ atm and $a_0 = 3.4 \times 10^4$ cm/sec), the blast wave has traveled a distance of 1.05 cm by the end of the laser pulse. Hence the assumption of planar blast wave appears to be good.

Now the surface pressure, as derived by Pirri,[9] can be written simply as:

$$p_s = \left[\frac{(\gamma+1)}{2\gamma}\right]^{2\gamma/(\gamma-1)} \cdot \frac{\rho_0}{\gamma+1}\left(\frac{Ia_0^2}{p_0}\right)^{2/3} = 0.16\,\rho_0\left(\frac{Ia_0^2}{p_0}\right)^{2/3} \tag{1}$$

where γ is the ratio of specific heats and ρ is the density. In this expression, $\gamma = 1.2$ has been assumed. Within this model, the pressure on the surface is constant when the laser beam is on. For intensities of 10^7 and 10^8 W/cm^2, the pressure takes on a value of 49 and 226 atm, respectively.

The blast wave is assumed to continue its one-dimensional character until it reaches a distance of $d/2$. Then it is assumed to decay as a spherical blast wave. For $\tau \leq t \leq t_3$ where t_3 is the time of transition from one- to three-dimensional expansion, the one-dimensional similarity solution of the blast wave theory requires $R \sim t^{2/3}$ and $p_s \sim t^{-2/3}$. With the constant evaluated at $t = \tau$, one has:

$$R = (Ia_0^2\tau/p_0)^{1/3}\, t^{2/3}$$

$$p_s = 0.16\,\rho_0 (Ia_0^2\tau/p_0)^{2/3}\, t^{-2/3} \tag{2}$$

The transition time t_3 can be evaluated from the fact that at $t = t_3$, $R = d/2$

$$t_3 = (d^3 p_0/8\, Ia_0^2\tau)^{1/2}$$

Finally, for $t > t_3$, the wave will decay to ambient pressure as a spherical blast wave which requires $R \sim t^{2/5}$ and $p \sim t^{-6/5}$. Satisfying the continuity condition at t_3, one gets:

$$p_s = 0.16\,\rho_0 (Ia_0^2\tau/p_0)^{2/3}\, t_3^{8/15}\, t^{-6/5} \tag{3}$$

Equations (1-3) provide pressure distributions for high intensity laser beams. As shown in Fig. 1, the above model has been applied to intensities 10^7 and 10^8 W/cm^2. The pressure is constant for $t \leq \tau$. After the laser beam is terminated, the pressure decays as a classical planar blast wave. For $I = 10^7$ and 10^8 W/cm^2, the one-dimensional pressure decay brings the pressure to less than 10% of the peak at the transition point of $t = 1.7 \times 10^{-4}$ and 5.5×10^{-5} sec, respectively. The pressure subsequently decays as a spherical blast wave until it reaches the ambient pressure. An alternative means of

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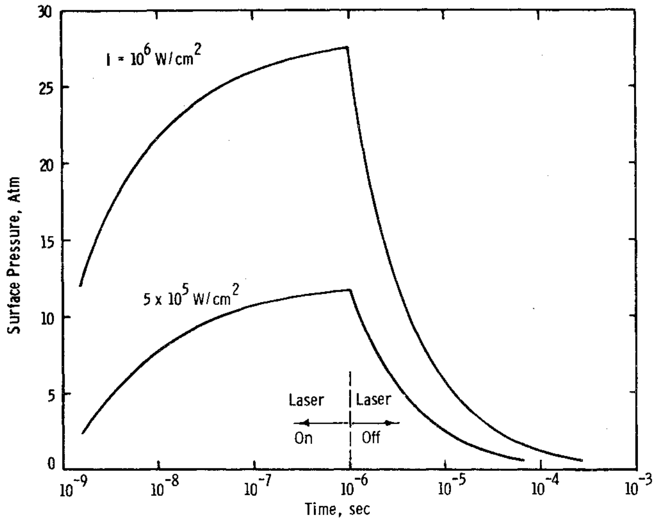

Fig. 2 Surface pressure as function of time for low intensity laser pulses.

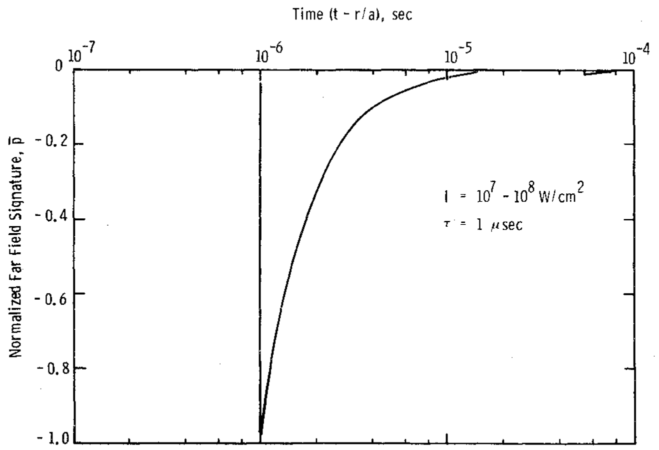

Fig. 3 Normalized far-field signature as function of time for high intensity laser pulses.

selecting the transition point, i.e., from one- to three-dimensional expansion, is to use the classical theory for the spherical blast wave with an instantaneous energy input in an infinitesimally small region of the medium for the late-time behavior. A sample calculation was carried out for a laser intensity of 10^8 W/cm^2. As shown in Fig. 1, the selection of the transition point at $R = d/2$ has not been critical.

Low Intensity Regime

Blast wave theory is appropriate when the laser pulse is short and the overpressure is large compared to one atmosphere, i.e., for high irradiance $10^7 - 10^8$ W/cm^2. When the overpressure is not large compared to one atmosphere, a more detailed theory must be used. Here we used the previously developed conduction code[3] to compute the surface pressure histories for low irradiances. This code contains the following assumptions: 1) the laser radiation is assumed to be absorbed at the surface, 2) the vapor on the surface obeys the equilibrium vapor pressure curve of water, and 3) after the termination of the laser pulse, the vapor pressure is assumed to decay as a planar blast wave. The first assumption requires that the optical absorption depth be small compared with the thermal depth.

Calculations have been done for intensities of 10^5, 5×10^5, and 10^6 W/cm^2 and for 1 μsec pulses. The surface temperatures rise rapidly to a constant value and remain approximately constant until the termination of the laser pulse. After the terminations of a laser pulse one can integrate the

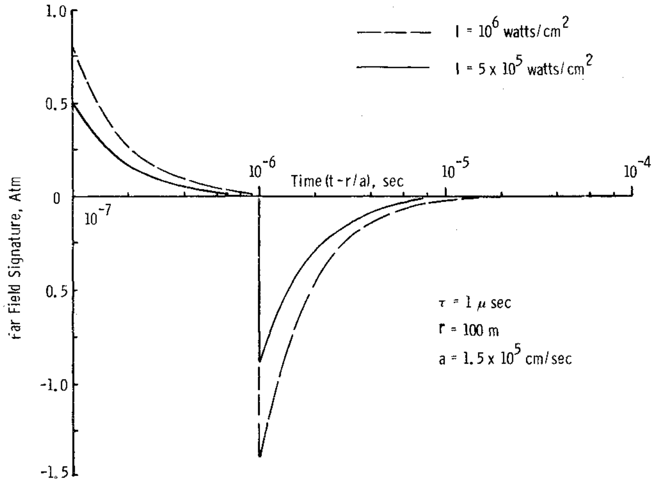

Fig. 4 Far-field signature as function of time for low intensity laser pulses.

temperature profile in the water to obtain the residual energy in the water. For intensities of 10^5, 5×10^5, and 10^6 W/cm^2, the energy fractions remaining in the water are 18, 6, and 4%. The corresponding vapor pressure histories for 5×10^5 and 10^6 W/cm^2 are shown in Fig. 2. The vapor pressure for the case of 10^5 W/cm^2 is too low to be of interest, i.e., about 4 atm.

Far-Field Signature

When a laser beam strikes the water surface, high pressure pulse is delivered to the water which is a relatively incompressible fluid. The bulk modulus of elasticity of water, which is about 20,000 atm,[10] implies that an overpressure of 1000 atm is required to create a density variance, $\Delta\rho/\rho$, of 0.05. For the intensity of 10^8 W/cm^2, the overpressure is approximately 226 atm and hence a "weak" shock wave will be generated in the water. From linear acoustic theory,[11] the far-field signature normal to the source due to a surface pressure pulse $p_s(t)$ can be written as:

$$p\left(r,t+\frac{r}{a}\right) = \frac{1}{2\pi ra}\int\int_A \frac{\partial p_s(x,y,t)}{\partial t}\,\mathrm{d}x\,\mathrm{d}y \qquad (4)$$

where r is the distance from the source to the observation, and a is the sound speed in water. For the high intensity regime, i.e., $10^7 - 10^8$ W/cm^2, the far-field signature becomes a delta function at $t = r/a$ and again a negative delta function at $t = r/a + \tau$, where τ is the pulse time. For $t > r/a + \tau$

$$p\left(r,t+\frac{r}{a}\right) = \frac{1}{2\pi ra}\frac{\partial}{\partial t}\left[0.16\rho_0\left(\frac{Ia_0^2\tau}{p_0}\right)^{2/3} t^{-2/3}\right] x\frac{\pi d^2}{4}$$

$$= -\phi(I,r)\,(t/\tau)^{-5/3}$$

where the amplitude function

$$\phi = \frac{0.013\,\rho_0 d^2}{ra}\left(\frac{Ia_0^2\tau}{p_0}\right)^{2/3}$$

The time history of the normalized far-field signature, $\bar{p} = -(t/\tau)^{-5/3}$, is shown in Fig. 3. Again, assuming 1 μsec pulse, $r = 100$ m, and a spot size diameter of 30.48 cm, one can readily evaluate the amplitude function, $\phi(I,r)$, and for $I = 10^7$ and 10^8 W/cm^2, ϕ takes on the values of 2.5 and 11.7 atm, respectively, at $t = r/a + \tau$.

For the low-intensity regime, the far-field signature can be obtained by inserting the surface pressure histories in Fig. 2 into Eq. (4) and numerically evaluating the integral. For $\tau = 1$

μsec and $r = 100$ m, the results are shown in Fig. 4. For $r/a < t < r/a + \tau$, the far-field signature, which has a finite amplitude, decreases with increasing time and approaches zero as $t \rightarrow r/a + \tau$. For $t > r/a + \tau$ the profile is the same as that in the high intensity regime and the maximum amplitude function for 5×10^5 W/cm^2 is 0.9 atm. These calculations have not taken into account the effects of frequency dependent absorption. The fast Fourier transform technique can be used to obtain such an attenuated signature.

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References

[1] Pirri, A.N., Schlier, R., and Northam, D., "Momentum Transfer and Plasma Formation Above a Surface with a High-Power CO_2 Laser," App. Phys. Letters, Vol. 21, No. 3, Aug. 1972, pp. 79-81.

[2] Wu, P.K., and Nebolsine, P.E., "Momentum Transfer to a Surface with a Pulsed Laser," AIAA Journal, Vol. 15, May 1977, pp. 751-753.

[3] Bunkin, F.V., Karlov, N.V., Komissarov, V.M., and Kuzmin, G.P., "Excitation of Sound When A Surface Layer Of A Liquid Absorbs A Laser Pulse," Soviet Physics – JETP Letters, Vol. 13, No. 9, 1971, pp. 341-343.

[4] Bell, C.E. and Maccabee, B.C., "Shock Wave Generation In Air and In Water By CO_2 TEA Laser Radiation," *Applied Optics,* Vol. 13, No. 3, 1974, pp. 605-609.

[5] Nebolsine, P.E., "Radiation-Induced Sound," PSI TR-82, Physical Sciences Inc., Woburn, Mass., 1977.

[6] Mason, B.J., *The Physics of Clouds,* 2nd Ed., Clarendon Press, Oxford, 1971.

[7] Reilly, J., Singh, P., and Weyle, G., "Multiple Pulse Laser Propagation through Atmospheric Dusts at 10.6 Microns," AIAA Paper 77-697, June 27-29, 1977.

[8] Dabora, E.K., "Variable Energy Blast Waves," AIAA Journal, Vol. 10, Oct. 1972, pp. 1384-1386.

[9] Pirri, A.N., "Theory for Momentum Transfer to a Surface with a High-Power Laser," *The Physics of Fluids,* Vol. 16, Sept. 1973, pp. 1435-1440.

[10] Streeter, V.L., *Fluid Mechanics,* McGraw Hill, New York, 1962.

[11] Morse, P.M. and Ingard, K.V., *Theoretical Acoustics,* McGraw Hill, New York, 1968.

Moving Thermal Contact Problems

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Introduction

ALTHOUGH the problem of moving thermal interfaces has received much attention for situations[1-3] such as melting, solidification, and ablation, little work has been directed toward moving contact problems. Such problems commonly occur in rolling mills, tire-road contact, etc. The lack of work in this area is largely an outgrowth of the fact that such thermal contact problems result in multidimensional, time-dpendent, mixed boundary-value formulations which are essentially analytically intractable. Such difficulties are further aggravated for media with temperature dependent properties.[4]

With the foregoing in mind, this Note will consider the finite element (FE) solution of moving thermal interface problems where there is a steady-state formation of the contact zone. Particular emphasis will be given to contact problems associated with either closed or "infinite" structures. To simplify the overall FE formulation, rather than employing numerical integration to handle the time dependency, the Galilean transform[4] is used to convert the governing field equations to a purely spatially dependent form. Such a transformation tends to freeze the contact formation in both space and time. This operation leads to a new type of FE formulation where the overall element "conductivity" (stiffness) is no longer symmetric. Regardless of the lack of symmetry, in addition to being more tractable, the solution to this formulation is not susceptible to the instabilities and convergence difficulties of the numerical integration approach. In the sections which follow, brief discussions will be given on the governing equations and FE development as well as the results of several numerical experiments.

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Governing Field Equations

For a structure composed of anisotropic temperature dependent media, the governing conduction field equation is defined by

$$\nabla \cdot ([K] \cdot \nabla T) + Q = \rho C_V \frac{\partial T}{\partial t} \tag{1}$$

where $[K]$ is the conductivity tensor, Q the heat generation, ρ the density, C_V the specific heat, T the temperature, t the time, and $\nabla \cdot (\ )$ and $\nabla (\ )$ are the divergence and gradient operators. Considering the problem of moving thermal contact, the boundary conditions associated with Eq. (1) take the form:

1) for all (x_1, x_2, x_3) on $S_{s\infty}(t)$

$$n_s \cdot [K_s] \cdot \nabla T_s = H_{s\infty}(T_s - T_\infty) \tag{2}$$

2) for all (x_1, x_2, x_3) on $S_{sg}(t)$

$$n_s \cdot [K_s] \cdot \nabla T_s = n_g \cdot [K_g] \cdot \nabla T_g \tag{3}$$

$$n_s \cdot [K_s] \cdot \nabla T_s = H_{sg}(T_s - T_g) \tag{4}$$

such that n_s, n_g are normals to bodies s and g; $H_{s\infty}$ is the convective coefficient for (x_1, x_2, x_3) on $S_{s\infty}(t)$; T_∞ is the ambient temperature in the vicinity of $S_{s\infty}(t)$ and, assuming imperfect contact, H_{sg} is the so-called contact conductance at the interface $S_{sg}(t)$.

Finite Element Development

Since Eq. (1) is time dependent and potentially nonlinear, the standard FE formulation will yield a system of first-order ordinary differential equations which must be solved numerically. Rather than handle the time dependency via direct numerical integration, since the steady-state case of moving thermal contact is assumed, the Galilean transform[4] can be used to reduce the problem to a purely spatially dependent nonlinear eigenvalue problem. As noted earlier, such an approach "freezes" the contact formation. Hence, the problem can be treated from a stationary point of view.

In particular, for the appropriate coordinate choice, the governing fields take the form namely

$$[T(x_1, x_2, x_3, t), \ldots] = [T(x_1, x_2, x_3 + \Omega t), \ldots] \tag{5}$$

where Ω is the speed of contact patch formation. For either x_3 infinite or closed† structures, the employment of the Galilean transform ($\xi = x_3 + \Omega t$) reduces Eq. (1) to the form

$$\nabla \cdot ([K] \cdot \nabla T) + Q = \rho \Omega C_V \frac{\partial}{\partial \xi}(T) \tag{6}$$

†For such a case, the governing fields are periodic in both space (x_3) and time.